

Hertentamen Kansrekening – Open book

Friday 23 August 2006

1. Let X be a random variable taking positive values with density $f_X(x)$.
Let $Y = \frac{1}{X+2}$. What is the density for Y ?

2. Let X and Y be independent random variables taking values in $\{1, 2, \dots\}$ with

$$P(X = i) = P(Y = i) = \frac{1}{2^i}$$

a) Use the formula $\sum_{j=0}^{\infty} q^j = \frac{1}{1-q}$ to check that this defines a probability weight function.

b) Show that $P(X = Y) = \frac{1}{3}$

c) Compute $P(\min(X, Y) \geq i)$

3. Let X, Y, Z be independent random variables, uniformly distributed on the interval $[0, 1]$.

a) Determine the density of the random variable XY .

b) Determine the density of the random variable XYZ .

4. Let A_n be an increasing sequence of events. (This means $A_n \subset A_{n+1}$ for all n). Denote $A = \bigcup_{n=1}^{\infty} A_n$. Let B be an event with $P(B) > 0$.

Show that $\lim_{n \rightarrow \infty} P(A_n|B) = P(A|B)$. *continuity prop.*

5. Let X and Y be independent Poisson random variables with parameters λ_X and λ_Y .

a) What is the distribution of $X + Y$? *Pois.*

b) What is the conditional distribution of X given $X + Y = k$? *bin.*

c) Can you formulate an extension of the results in a) and b) to n independent Poisson random variables? *mult.*